Unit 10 Study Guide: Plane Figures

Be sure to watch all videos within each lesson

You can find geometric shapes in art. Whether determining the amount of leading or the amount of glass needed for a piece of stained glass art, stained-glass artists need to understand perimeter and area to solve many practical problems.

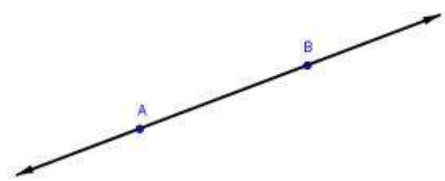
Lesson 1: Parallel Lines and Transversals

Point: a location in space with no length, width or depth

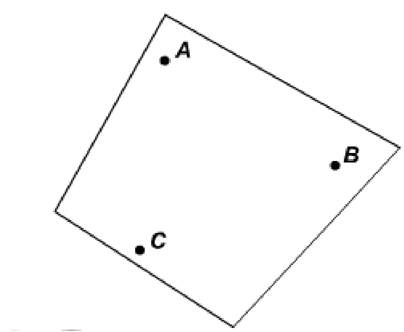
● M

Line: a collection of points arranged in a straight path that extends without end in both directions.

Line $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$

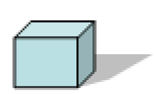


Plane: a flat surface with infinite length and width but no depth

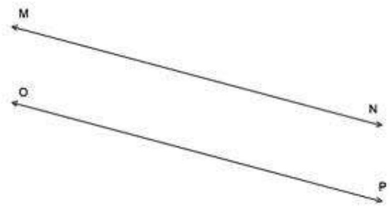


Plane ABC

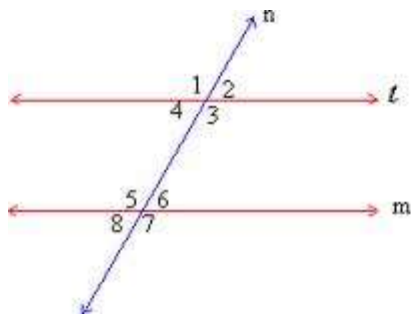
Plane *M*

point	line	Plane	Solid
Zero dimensions	One dimension	Two dimensions	Three dimensions
.			

Parallel Lines: Lines that are in the same plane that never intersect



Transversal: a line that intersects 2 or more lines in a plane



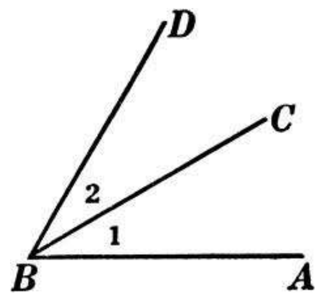
n is a transversal

t and m are parallel lines

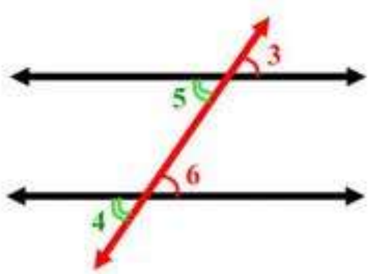
Adjacent Angles: 2 angles with a common vertex and a common side

***Adjacent angles are neighbors

<1 and <2 are adjacent angles



Corresponding Angles: angles that are in matching corners (they are on the same side of the transversal) ****Corresponding angles are congruent



<3 and < 6 are corresponding angles

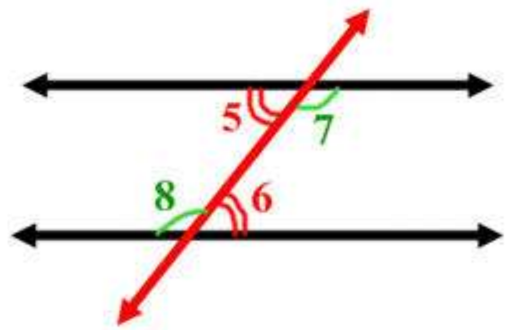
<5 and <4 are corresponding angles

Alternate Interior Angles: angles that are on opposite sides of the transversal and are between the parallel lines

<5 and <6 are Alternate Interior Angles

<7 and <8 are Alternate Interior Angles

****Alternate interior angles are congruent

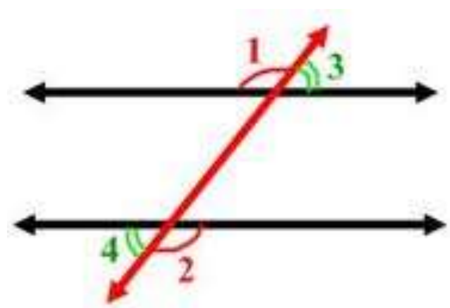


Alternate Exterior Angles: angles that are on opposite sides of the transversal and are outside of the parallel lines

<1 and <2 are Alternate Exterior Angles

<3 and <4 are Alternate Exterior Angles

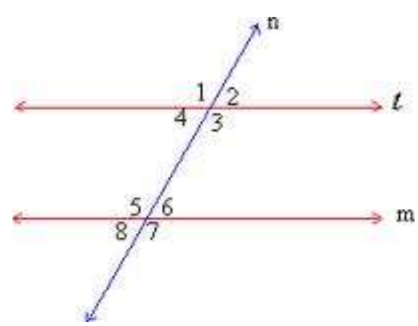
****Alternate exterior angles are congruent



Adjacent Angles are supplementary (their sum is 180^{o})

Adjacent angles:

<1 and <2	<5 and <6
<2 and <3	<6 and <7
<1 and <4	<7 and <8
<4 and <3	<8 and <5



Offline work:

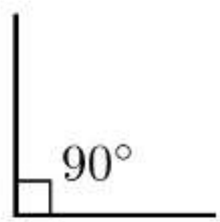
Read pages 313-315 in Reference Guide.
Complete Problems 1-17 odd on page 316
Complete Problems 2-16 even on page 316 for extra practice (optional)

Class Examples:

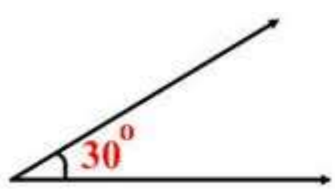
Lesson 2: Triangles

Types of Angles

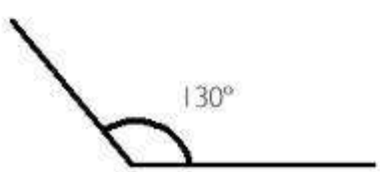
Right Angle: 90°



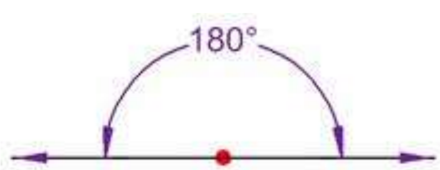
Acute Angle: less than 90°



Obtuse Angle: greater than 90°



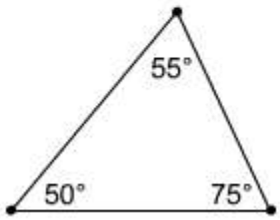
Straight Angle: 180°



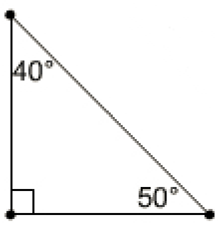
Triangle: a figure made up of 3 segments joined at their endpoints

Classifying Triangles by Angle Measures:

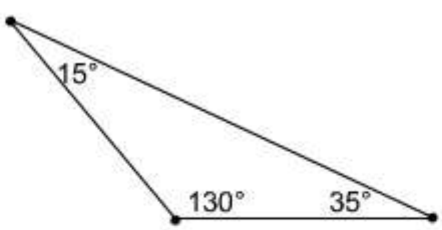
Acute Triangle: triangle with 3 acute angles

Example: 

Right Triangle: a triangle with exactly one right angle

Example: 

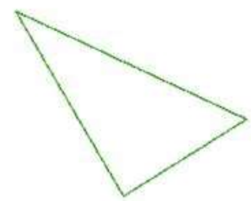
Obtuse Angle: a triangle with exactly one obtuse angle

Example: 

Triangle Angle Sum Property: when you add the measures of all 3 angles of a triangle, the sum is 180°.

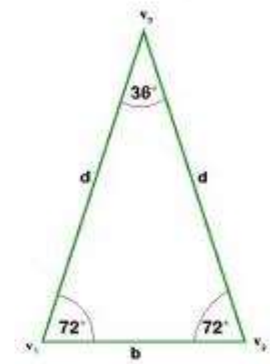
Classifying Triangles by the Sides:

Scalene: none of the sides are equal
(none of the angles are equal either)



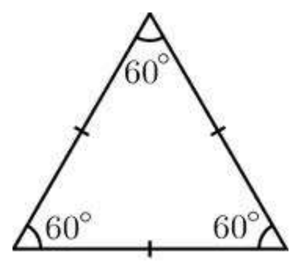
Isosceles: at least 2 of the sides are equal

Sides d are equal
Angles opposite sides d are equal



Equilateral: all 3 sides are equal

All 3 angles are equal



Offline Work:

Read pages 318-320 in Reference Guide.
Complete Problems 1-17 odd and 18 on pages 321-322.
Complete Problems 2-16 even on pages 321-322 for extra practice (optional).

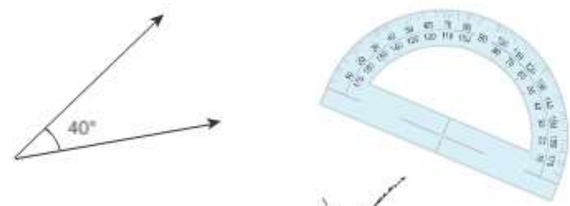
Lesson 3: Constructing Angles

You can use a protractor to measure the degrees in an angle.

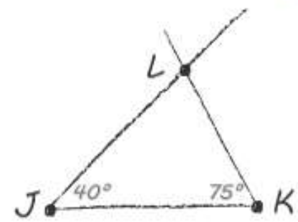
Watch the videos and the interactive activities to practice drawing angles and triangles.

Constructing Angles and Triangles

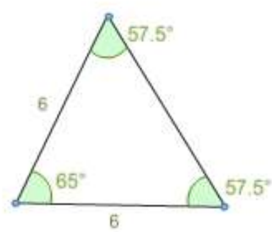
You can use a protractor to construct an angle with a given measure.



You can use a protractor to construct a triangle fitting a particular description.



You can use computer software to construct a triangle fitting a particular description.



<u>Offline Work:</u>

Read pages 323-326
Complete Problems 1-15 odd on page 326
Complete Problems 2-16 even on page 326 for extra practice (optional)

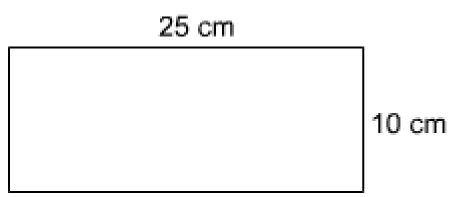
Lesson 5: Areas of Rectangles and Triangles

The interior of a figure is the space enclosed by the figure. The area of the figure is the number of square units in the interior of the figure.

Area of Rectangles – the formula for the area of a rectangle with base *b* and height *h* is A = *bh*

*The length and width of a rectangle are the same as the base and height, so the formula can also be written as A = *lw*

Find the area of the rectangle.



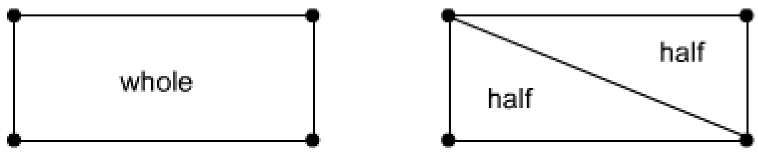
$A = lw$

$A = 25(10)$

$A = 250\ cm^2$

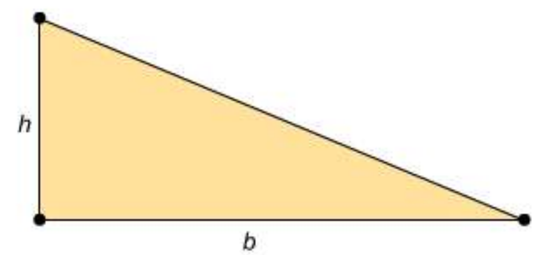
Area of Triangles

If you know the area of the rectangle, each of the two triangles will be half the area of the rectangle.



The area of a triangle is half the area of a rectangle with the same base and height.

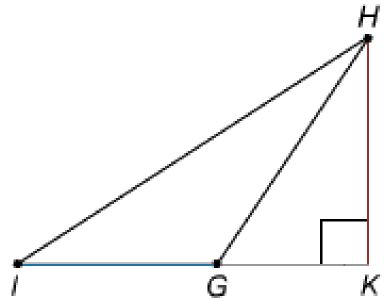
Area of a Triangle



$$A = \frac{1}{2}bh$$

The base and height of the triangle form a right angle. Any side can be called the base. The line segment that is perpendicular to the base is the height of the triangle.

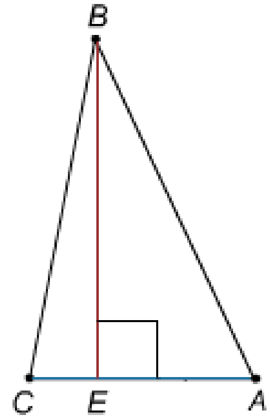
obtuse triangle



base = $\overline{IG}$

height = KH

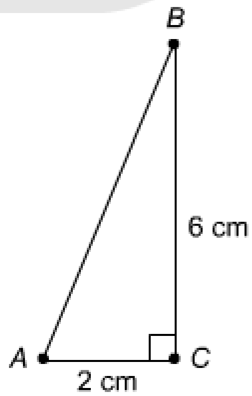
acute triangle



base = $\overline{CA}$

height = EB

What is the area of triangle *ABC*?



Apply the formula for the area of a triangle,

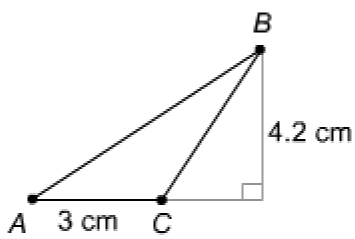
$A = \frac{1}{2}bh.$

The base is 2 cm, and the height is 6 cm.

$A = \frac{1}{2}bh = \frac{1}{2}(2)(6) = \frac{1}{2}(12) = 6$

The area of the triangle is 6 cm^2.

What is the area of triangle *ABC*?



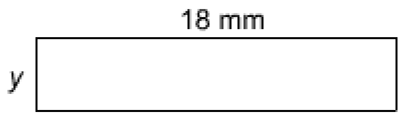
For this triangle, the base is 3 cm, and the height is 4.2 cm.

$A = \frac{1}{2}bh = \frac{1}{2}(3)(4.2) = \frac{1}{2}(12.6) = 6.3$

The area of the triangle is 6.3 cm^2.

Finding Missing Lengths - You can use the area formulas to find missing lengths of rectangles and triangles. Substitute known values into the appropriate formula, then solve for the missing length.

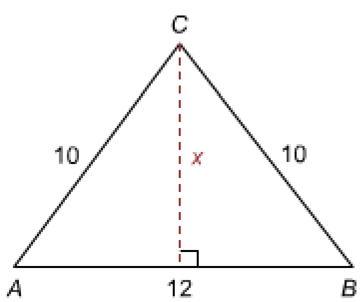
The area of the rectangle is 63 square millimeters. Find the missing length, *y*.



$A = lw$	Write the area formula.
$63 = 18y$	Substitute.
$3.5 = y$	Divide.

So the missing length $y = 3.5$ mm.

The area of the triangle is 48 square inches. Find the missing length, *x*.



$A = \frac{1}{2}bh$	Write the area formula.
$48 = \frac{1}{2}(12)x$	Substitute.
$48 = 6x$	Simplify.
$8 = x$	Divide.

So the missing length $x = 8$ inches.

Areas of Complex Figures

Dividing a more complex figure made up of rectangles or triangles (or both!) can help you find its area.

What is the area of *ABCJ*?

$AB = 4$ cm

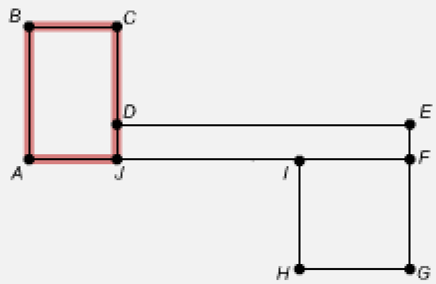
$BC = 2$ cm

$DJ = 1$ cm

$JF = 7$ cm

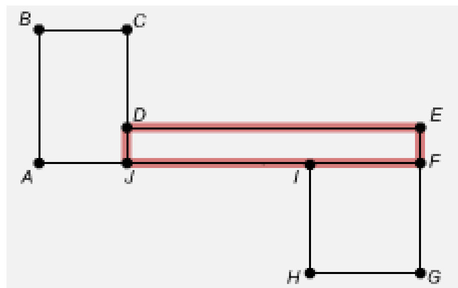
$IH = 3$ cm

$IJ = 4$ cm



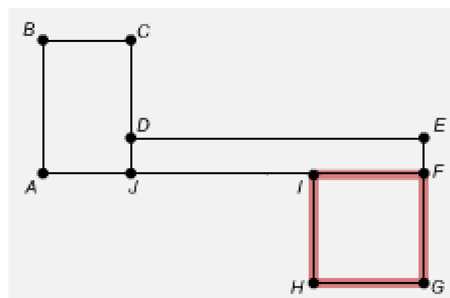
***AB* = 4 cm and *BC* = 2 cm**

The area of *ABCJ* = (4 cm)(2 cm) = 8 cm^2.



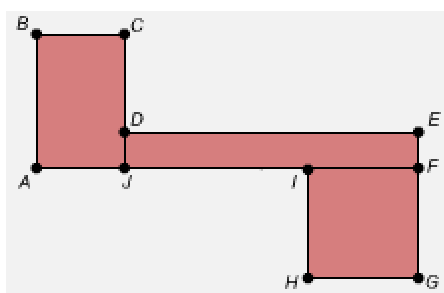
***DJ* = 1 cm and *JF* = 7 cm**

The area of *DEFJ* = (1 cm)(7 cm) = 7 cm^2.



We know that *IH* = 3 cm, but we have to figure out *IF*.
***JF* = 7 cm and *IJ* = 4 cm, so *IF* = 3**

The area *IFGH* = (3 cm)(3 cm) = 9 cm^2.



The area of the entire figure,
***A* = 8 cm^2 + 7 cm^2 + 9 cm^2 = 24 cm^2.**

<u>Offline Work:</u>

Read pages 327-330

Complete Problems 1-23 on pages 330-331

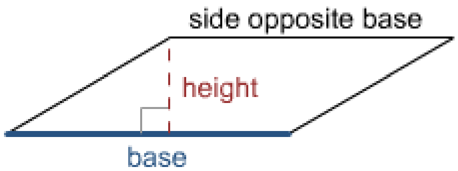
Lesson 6: Areas of Special Quadrilaterals

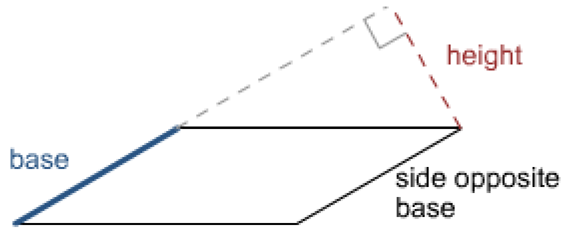
Area of a parallelogram = base × height

$$A = bh$$

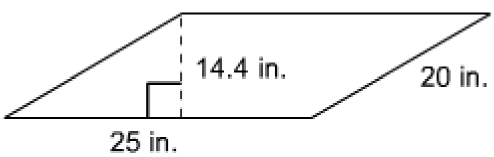
the base of the figure (and one of its four sides)

the height of the parallelogram (perpendicular to the base)





Find the area of the parallelogram:



A = bh

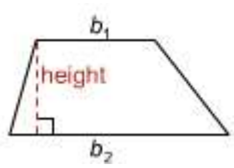
A = 25 x 14.4

A = 360 in.^{2}

Area of Trapezoids

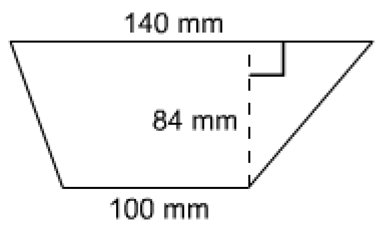
Formula for area of a trapezoid:

$A = \frac{1}{2}(b_1 + b_2)h$ or $\frac{1}{2}h(b_1 + b_2)$



b_1 and b_2 are the bases – the parallel sides
h is the height of the trapezoid, perpendicular to the bases

Find the area of this trapezoid.



The sum of the bases of the trapezoid is (140 + 100) mm and the height is 84 mm.

$A = \frac{1}{2}(b_1 + b_2)(h) = \frac{1}{2}(140 + 100)(84) = \frac{1}{2}(20{,}160) = 10{,}080$

The area of the trapezoid is 10,080 mm^2.

Offline work:

Read pages 333-335 in Reference Guide
Complete Problems 1-21 odd on pages 336-337
Complete Problems 2-22 even on pages 336-337 for extra practice (optional)

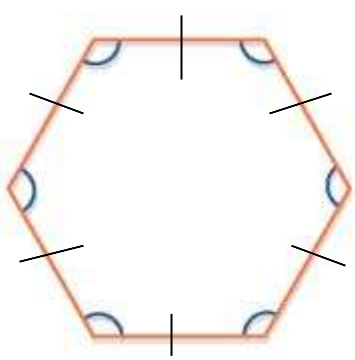
Lesson 7: Areas of Polygons

Polygon –a 2-dimensional figure with 3 or more straight sides. Each side meets only at its endpoints, called a vertex.

Equiangular Polygon – all the angles are equal

Equilateral Polygon – all the sides are equal

Regular Polygon – all the sides are equal and all the angles are equal

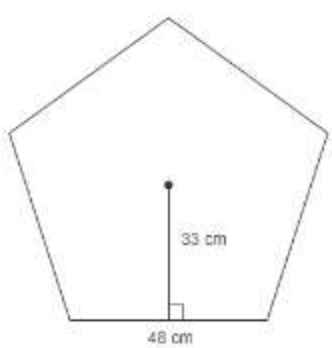


The lines through the sides tell you the sides are equal
The half circles marking the angles tell you the angles are equal

Classifying Polygons:

Number of Sides	Polygon Name
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
10	Decagon
n	n-gon

The **apothem** of a regular polygon is a line segment drawn from the center of the polygon to one of its sides. It is perpendicular to that side.



To find the area of a regular polygon:

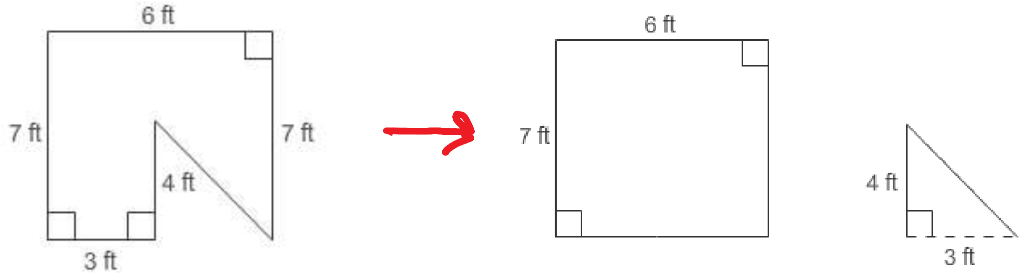
Step 1: Divide the figure into congruent triangles.

Step 2: Find the area of 1 of the triangles, using the apothem as the height.

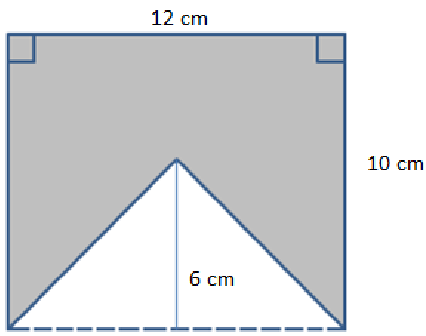
Step 3: Multiply the area of this 1 triangle by *n*, the number of congruent triangles, which is equal to the number of sides of the polygon.

Finding the Area of an Irregular Polygon

Sometimes, it is easier to represent an irregular polygon as another polygon with 1 or more familiar polygons removed.



What is the area of the shaded figure?



First find the area of the entire rectangle. A = bh

$A = 12 \times 10$

$A = 120\ cm^2$

Second, find the area of the triangle. $A = \frac{1}{2}bh$

$A = \frac{1}{2}(12)(6)$

$A = \frac{1}{2}(72)$

$A = 36\ cm^2$

Third, subtract the area of the triangle from the area of the rectangle.

$120cm^2 - 36cm^2 = 84cm^2$

<u>Offline work:</u>

Read pages 338-341 in Reference Guide

Complete problems 1-11 on pages 341-342

<u>CLASS EXAMPLES</u>:

Lesson 8: Core Focus: How Many Triangles?

The Core Concept

You can determine how many triangles fit a given description by applying some triangle math facts.

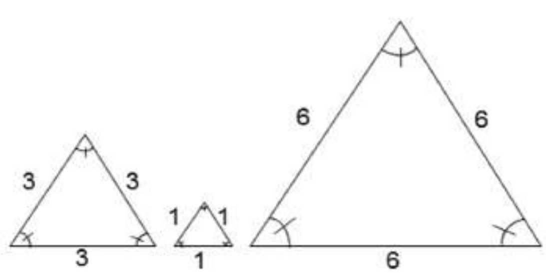
Strategy 1: Using Angle Measures

Math Fact #1: The sum of all 3 angle measures in a triangle is equal to 180°.

If the sum of the 3 angle measures does not add up to exactly 180°, then no triangles are possible.

Math Fact #2: Given 3 angles whose measures sum to 180°, an infinite number of triangles are possible.

An infinite number of triangles can be constructed.

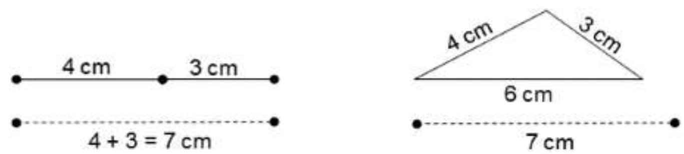


Strategy 2: Using Side Lengths

Math Fact #3: The longest side of a triangle is always shorter than the other 2 side lengths put together.

- If the longest side is shorter than the sum of 2 other side lengths, then exactly 1 triangle is possible.
- If the longest side is longer than or equal to the sum of the 2 other side lengths, then no triangle is possible.

$3 + 4 = 7 > 6$



Strategy 3: Working Through By Hand

When given a combination of side lengths and angle measures, you often have to work through different possibilities for triangle shapes.

- Use paper, a pencil, a ruler, and a protractor to work through examples.
- Start with 1 side length.

<u>Offline Work:</u>

Read pages 343-345 in Reference Guide

Complete Problems 1-3 on page 345