Study Island Guide – 7.5 March Pathways

PROBABILITY

*If an event has a probability close to 0, the event is **unlikely** to occur.*

*If an event has a probability close to $\frac{1}{2}$, the event is **neither unlikely nor likely** to occur.*

*If an event has a probability close to 1, the event is **likely** to occur.*

Example 1:

The probability of selecting a red chip from a bag of plastic chips is $\frac{9}{16}$. Describe the likelihood of selecting a red chip.

Solution:

Since the probability is close to $\frac{1}{2}$, it is about the same distance from both 0 and 1.

Therefore, the likelihood of selecting a red chip is **neither unlikely nor likely**.

Example 2:

The probability of a spinner landing on a yellow section is $\frac{2}{11}$. Describe the likelihood of the spinner landing on a yellow section.

Solution:

Since the probability is closer to 0 than to $\frac{1}{2}$, the likelihood of the spinner landing on a yellow section is **unlikely**.

Example 3:

The probability of selecting a blue marble from a bag of marbles is $\frac{9}{10}$. Describe the likelihood of selecting a blue marble.

Solution:

Since the probability is closer to 1 than to $\frac{1}{2}$, the likelihood of selecting a blue marble is **likely**.

Probability *refers to the chance that an event will happen.*

Complement *refers to the chance that an event will not happen.*

$P' = 1 - P$

Probability can be presented as a ratio of the number of ways an event can occur relative to the number of possible outcomes.

$$\textit{Probability of Event} = \frac{\textit{Number of ways event can occur}}{\textit{Number of possible outcomes}}$$

Example 1

When rolling a die, what is the probability of rolling a four?

Solution

$$\text{P(rolling a 4)} = \frac{\text{Number of ways rolling a four can occur}}{\text{Number of possible rolls}} = \frac{1}{6}$$

Example 2

When rolling a die, what is the probability of rolling a number less than 4?

Solution

$$\text{P(rolling less than a 4)} = \frac{\text{Number of ways rolling less than four can occur}}{\text{Number of possible rolls}} = \frac{3}{6} = \frac{1}{2}$$

Example 3

When picking from a bag that contains 5 blue marbles, 2 green marbles, and 3 red marbles, what is the probability of picking a red marble?

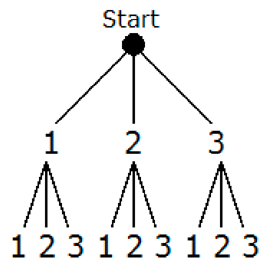
Solution

$$\text{P(red marble)} = \frac{\text{Number of ways picking a red marble can occur}}{\text{Number of possible picks (5 blue + 2 green + 3 red)}} = \frac{3}{10}$$

*A **tree diagram** can be used to find the probability of two events.*

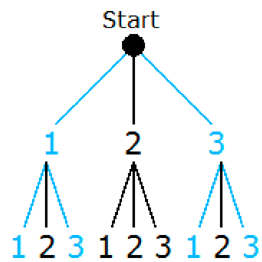
Example:

Kyndall spins a spinner with three equal sections, numbered 1 to 3, twice. What is the probability that Kyndall spins two odd numbers?



Solution:

Find each branch that has two odd numbers. Of the numbers on the spinner, 1 and 3 are odd.



There are 4 branches out of a total of 9 branches which have two odd numbers.

*Sometimes when finding the probability of an event, it is helpful to **make a list** of all the possible outcomes. This is a good option when the number of possible outcomes is small.*

Example:

A small bin of highlighters has 5 yellow, 2 purple, 2 green, and 1 orange highlighter. If Mandi chooses two highlighters randomly, what is the probability that she chooses a purple highlighter and an orange highlighter?

Solution:

In this situation, it is helpful to make a list of all the possible combinations.

1. yellow, yellow	5. purple, yellow	9. green, yellow	13. orange, yellow
2. yellow, purple	6. purple, purple	10. green, purple	14. orange, purple
3. yellow, green	7. purple, green	11. green, green	15. orange, green
4. yellow, orange	8. purple, orange	12. green, orange	

So, there are 15 different color combinations of highlighters possible. Out of the 15 possible combinations, there are two combinations where Mandi chooses a purple highlighter and an orange highlighter.

1. yellow, yellow	5. purple, yellow	9. green, yellow	13. orange, yellow
2. yellow, purple	6. purple, purple	10. green, purple	14. **orange, purple**
3. yellow, green	7. purple, green	11. green, green	15. orange, green
4. yellow, orange	8. **purple, orange**	12. green, orange	

The probability that Mandi chooses a purple highlighter and an orange highlighter is equal to the number of ways she can choose a purple highlighter and an orange highlighter over the total number of outcomes.

Therefore, the probability Mandi chooses a purple highlighter and an orange highlighter is $\frac{2}{15}$.

To predict the number of times an event will occur based on its theoretical probability, multiply the number of trials of the experiment by the probability of the event occurring.

Example:

A spinner has five equally-sized sections labeled 1 through 5. If the spinner is spun 60 times, what is the best prediction of the number of times the spinner will land on section 1.

Solution:

First, find the theoretical probability that the spinner will land on section 1. The spinner has five equally-sized sections, so the probability of the spinner landing on section 1 is $\frac{1}{5}$.

Next, to predict the number of times the spinner will land on section 1, multiply 60 by the theoretical probability.

$$60 \times \frac{1}{5} = 12$$

Remember that this is an estimate instead of an exact value. Therefore, the best prediction of the number of times the spinner will land on section 1 is **12 times**.

Comparing Statistics

*By observing and comparing statistics of two different samples, a **generalization** can be made about the two sets of data.*

Example:

A middle school principal randomly selected 40 seventh-grade students, 20 males and 20 females, and measured their heights in inches.

	Males	Females
Mean	64	68
Median	62	65
Mode	63	66
Range	10	12

Based on these samples, what generalization can be made?

A. More males are 65 inches or taller than females that are 65 inches or taller.

B. More females are 65 inches or taller than males that are 65 inches or taller.

C. All of the male students are taller than all of the female students.

D. All of the female students are taller than all of the male students.

Solution:

The median is the statistic which can help draw conclusions relative to half of the data in the sample.

The median height of the female students is 65 inches.
So, at least half of the female students (at least 10) are 65 inches or taller.

The median height of the male students is 62 inches.
So, less than half of the male students (less than 10) are 65 inches or taller.

Therefore, the generalization, based on this sample, is that **more females are 65 inches or taller than males that are 65 inches or taller**.

*The **mean absolute deviation** can be used to measure **variability** in a set of data. It is the mean of the differences of the values in the data set from the mean.*

***Comparing statistics** can also include comparing means, medians, or modes of two data sets to the mean absolute deviation of those two data sets.*

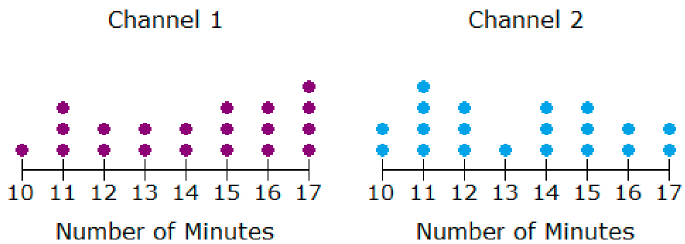
***mean**: the sum of all values in the data set divided by the number of values in the data set*

***median**: the middle value of a data set with an odd number of values or the average of the two middle values in a data set with an even number of values*

***mode**: the value which occurs most frequently in a data set*

Example:

The dot plots below show the number of minutes of advertising during different 60-minute shows on two television channels.



The mean absolute deviation for each month is 2. The difference between the mode number of minutes of advertising for each channel is how many times the mean absolute deviation?

Solution:

The mode is the number that occurs the most often.

The mode number of minutes of advertising for channel 1 is 17 minutes.
The mode number of minutes of advertising for channel 2 is 11 minutes.

Therefore, the difference between the mode number of minutes of advertising for each channel is 17 - 11 = 6.

Divide this difference, 6, by the mean absolute deviation, 2.

$$\frac{6}{2} = 3$$

So, the difference between the mode number of minutes of advertising for each channel is **3** times the mean absolute deviation.

Sampling Analysis

*In a **sample**, some members of the population are selected and considered representative of the entire population.*

Population - A population is an entire group of persons or elements that have at least one thing in common (Minnesota fourth graders, Moorhead State University summer school students).

Sample - A sample is a group of persons or elements selected from the total population.

Random sampling - Random sampling is where each member of the sample is randomly selected from the population.

Sampling bias - Sampling bias is caused by systematic errors in the sampling process. For example, take one-fourth of the students in a class as a sample to use in a research study. Send out notes to the parents requesting permission for their child to participate in the study and then select those students whose parents give permission first as the sample for the study.

Sample size - In general, the larger the sample size, the more representative it is of the population.

Example:

Chester wants to find out the size of the tomatoes in his large tomato garden. He decides to tag and measure 20 tomatoes from the garden. What would be the best way for Chester to ensure he gets a random sample?

Solution:

In order to collect a sample that is representative of the entire garden, **Chester should tag and measure 20 tomatoes at randomly chosen locations across the garden**.

*Characteristics of a representative **sample**, such as its mean, median, or mode, can be used to predict the mean, median, or mode of the entire **population**.*

Example:

An event center, which is divided into 25 same-sized sections, hosted a sold-out rock concert. To get an idea of how many ticket holders did not show up for the concert, the event planner randomly chose 10 sections and counted the number of empty seats in those sections during the concert, as shown below.

12, 8, 5, 10, 6, 3, 7, 14, 9, 6

Assuming the sample was representative of the entire event center, what was the mean number of ticket holders per section who did not show up for the concert?

Solution:

Since the sample is representative of all of the sections, the mean of the sample can be extended to all of the sections.

$$\begin{aligned} \text{mean} &= \frac{12+8+5+10+6+3+7+14+9+6}{10} \\ &= \frac{80}{10} \\ &= 8 \end{aligned}$$

Therefore, the mean number of ticket holders per section who did not show up for the concert was **8** ticket holders.