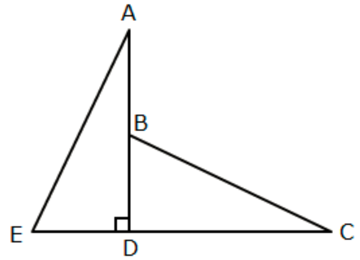
Angles

. In the figure below, triangle ADE is congruent to triangle CDB.



Note: picture not drawn to scale

If m∠DAE = 23°, what is m∠ABC?

A 157°

B 67°

C 103°

D 113°

Explanation

Since ∠EDA is a right angle, m∠EDA = 90°.

The sum of the angle measures in a triangle always equals 180°.

$$m\angle DAE + m\angle EDA + m\angle AED = 180^\circ$$
$$23^\circ + 90^\circ + m\angle AED = 180^\circ$$
$$113^\circ + m\angle AED = 180^\circ$$
$$m\angle AED = 67^\circ$$

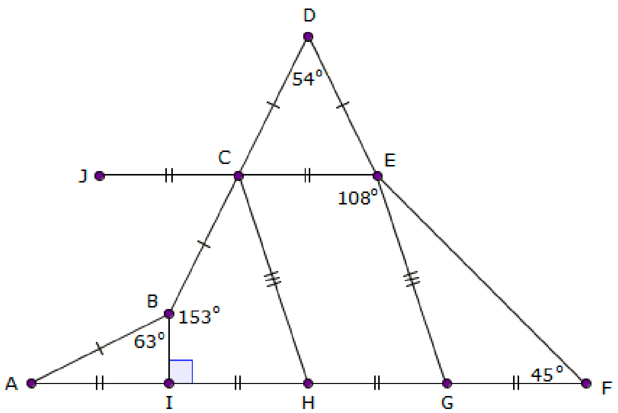
Since triangle ADE is congruent to triangle CDB, corresponding sides and corresponding angles are congruent. So, m∠AED = m∠CBD = 67°.

Since ∠CBD and ∠ABC form a linear pair, they are supplementary angles and the sum of their measures is 180°.

$$m\angle CBD + m\angle ABC = 180^\circ$$
$$67^\circ + m\angle ABC = 180^\circ$$
$$m\angle ABC = 113^\circ$$

Therefore, m∠ABC = **113°**.

. **Directions: Drag the tiles to the boxes to form correct pairs. Not all tiles will be used.**



Using the image shown above, match the angles that have the same measure.

$\measuredangle$GEF	$\measuredangle$JCB	$\measuredangle$CDE	$\measuredangle$EFG	$\measuredangle$DEC	$\measuredangle$HCE	$\measuredangle$IAB
$\measuredangle$CHI	$\measuredangle$BCH					

So, the measure of angle DCE is 63° and the measure of angle DEC is 63°.

Since angle JCB and angle DCE are vertical angles, the measure of angle JCB is 63°.

Therefore, $\measuredangle$DEC and $\measuredangle$JCB have the same measure.

Quadrilateral HCEG is a parallelogram and the measure of angle CEG is 108°. Since angle GHC is opposite angle CEG and opposite angles of a parallelogram have the same measure, the measure of angle GHC is 108°. Since angle HCE is consecutive to angle CEG in quadrilateral HCEG and consecutive angles of a parallelogram are supplementary, the measure of angle HCE can be found as follows.

$$\text{m}\measuredangle HCE + \text{m}\measuredangle CEG = 180^\circ$$
$$\text{m}\measuredangle HCE + 108^\circ = 180^\circ$$
$$\text{m}\measuredangle HCE = 180^\circ - 108^\circ$$
$$\text{m}\measuredangle HCE = 72^\circ$$

So, the measure of angle HCE is 72°.

Similarly, since angle EGH is consecutive to angle CEG in Quadrilateral HCEG and consecutive angles of a parallelogram are supplementary, the measure of angle EGH can be found as follows.

$$\text{m}\measuredangle EGH + \text{m}\measuredangle CEG = 180^\circ$$
$$\text{m}\measuredangle EGH + 108^\circ = 180^\circ$$
$$\text{m}\measuredangle EGH = 180^\circ - 108^\circ$$
$$\text{m}\measuredangle EGH = 72^\circ$$

So, the measure of angle EGH is 72°.

Angle GHC and angle CHI are supplementary angles. Since the measure of angle GHC is 108°, the measure of angle CHI is 180° - 108° = 72°.

Therefore, $\measuredangle \text{HCE}$ and $\measuredangle \text{CHI}$ have the same measure.

Angles JCB, BCH, and HCE form a straight line. Since the measures of angles JCB and HCE are known, find the measure of angle BCH.

$$\begin{aligned} \text{m } \measuredangle JCB + \text{m } \measuredangle BCH + \text{m } \measuredangle HCE &= 180^\circ \\ 63^\circ + \text{m } \measuredangle BCH + 72^\circ &= 180^\circ \\ \text{m } \measuredangle BCH &= 180^\circ - 63^\circ - 72^\circ \\ \text{m } \measuredangle BCH &= 45^\circ \end{aligned}$$

It is given in the image that the measure of angle EFG is 45°.

Therefore, $\measuredangle \text{EFG}$ and $\measuredangle \text{BCH}$ have the same measure.

Angle FGE and angle EGH are supplementary angles. Since the measure of angle EGH is 72°, the measure of angle FGE is 180° - 72° = 108°. The sum of the interior angles of a triangle is 180°. Find the measure of angle GEF.

$$\begin{aligned} \text{m } \measuredangle GEF + \text{m } \measuredangle EFG + \text{m } \measuredangle FGE &= 180^\circ \\ \text{m } \measuredangle GEF + 45^\circ + 108^\circ &= 180^\circ \\ \text{m } \measuredangle GEF &= 180^\circ - 45^\circ - 108^\circ \\ \text{m } \measuredangle GEF &= 27^\circ \end{aligned}$$

Triangle BIA is a right triangle. Since the two non-right angles of a right triangle are complementary and it is given that the measure of angle ABI is 63°, the measure of angle IAB can be found as follows.

$$\begin{aligned} \text{m } \measuredangle IAB + \text{m } \measuredangle ABI &= 90^\circ \\ \text{m } \measuredangle IAB + 63^\circ &= 90^\circ \\ \text{m } \measuredangle IAB &= 90^\circ - 63^\circ \\ \text{m } \measuredangle IAB &= 27^\circ \end{aligned}$$

So, the measure of angle IAB is 27°.

Therefore, $\measuredangle \text{IAB}$ and $\measuredangle \text{GEF}$ have the same measure.

Triangles

. How many triangles exist with the given side lengths?

3 cm, 5 cm, 9 cm

A More than one triangle exists with the given side lengths.

B Exactly one unique triangle exists with the given side lengths.

C No triangle exists with the given side lengths.

Explanation

According to the triangle inequality theorem, the sum of any two sides must be greater than the third side.

Since the sum of 3 cm and 5 cm is not greater than 9 cm, the side lengths do not form a triangle.

Therefore, **no triangle exists with the given side lengths**.

. How many triangles exist with the given angle measures?

60°, 60°, 60°

Exactly one unique triangle exists with the given angle measures.

More than one triangle exists with the given angle measures.

No triangle exists with the given angle measures.

Explanation

The sum of the angles in a triangle is 180°, so the angles do form a triangle.

Since all three angles are congruent, the triangle is an equilateral triangle. An infinite number of equilateral triangles exist with different side lengths.

Therefore, **more than one triangle exists with the given angle measures**.

How many triangles exist with the given angle measures?

55°, 45°, 90°

Exactly one unique triangle exists with the given angle measures.

More than one triangle exists with the given angle measures.

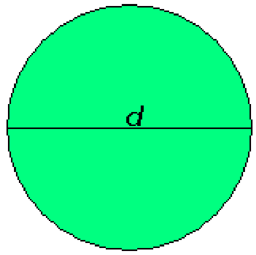
No triangle exists with the given angle measures.

Explanation

The sum of the angles in a triangle is 180°. These angles add up to more than 180°, so the angles do not form a triangle.

Therefore, **no triangle exists with the given angle measures**.

Circles



The diameter of the circle above is 26 in. What is the circumference of the circle?
Use π = 3.14.

$$\text{Circumference} = 2\pi r$$

A 40.82 in

B 81.64 in

C 1,061.32 in

D 530.66 in

Explanation

First, find the radius of the circle.

$$\begin{aligned} r &= d \div 2 \\ &= 26 \text{ in} \div 2 \\ &= 13 \text{ in} \end{aligned}$$

Next, use the formula for circumference.

$$\begin{aligned} \text{Circumference} &= 2\pi r \\ &= 2(3.14)(13 \text{ in}) \\ &= 81.64 \text{ in} \end{aligned}$$

- The area of circular garden is 379.94 ft^2. What is the circumference of the garden? Use π = 3.14.

$$\text{Circumference} = 2\pi r$$
$$\text{Area} = \pi r^2$$

A 189.97 ft

B 69.08 ft

C 126.65 ft

D 34.54 ft

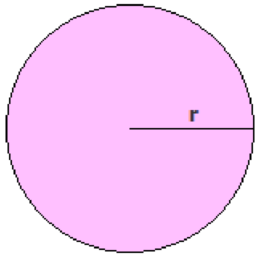
Explanation

First, use the area formula and the area to find the radius, *r*.

$$\text{Area} = \pi r^2$$
$$379.94\ \text{ft}^2 = 3.14 r^2$$
$$\frac{379.94\ \text{ft}^2}{3.14} = r^2$$
$$121\ \text{ft}^2 = r^2$$
$$11\ \text{ft} = r$$

Next, use the circumference formula and the radius to find the circumference of the garden.

$$\text{Circumference} = 2\pi r$$
$$= 2(3.14)(11\ \text{ft})$$
$$= 69.08\ \text{ft}$$



If the circumference of the circle above is 31.4 units, what is the area?
Use π = 3.14.

$$\text{Circumference} = 2\pi r$$
$$\text{Area} = \pi r^2$$

15.7 units^2

78.5 units^2

985.96 units^2

62.8 units^2

Explanation

First, use the circumference formula and the circumference to find the radius, *r*.

$$\begin{aligned} \text{Circumference} &= 2\pi r \\ 31.4 \text{ units} &= 2(3.14)r \\ 31.4 \text{ units} &= 6.28r \\ \frac{31.4 \text{ units}}{6.28} &= r \\ 5 \text{ units} &= r \end{aligned}$$

Next, use the area formula and the radius to find the area.

$$\begin{aligned} \text{Area} &= \pi r^2 \\ &= 3.14(5 \text{ units})^2 \\ &= 3.14(25 \text{ units}^2) \\ &= 78.5 \text{ units}^2 \end{aligned}$$

Directions: Type the correct answer in each box. Use numerals instead of words. Use 3.14 for π.

Jenna is planting four circular gardens. She is planting one garden each for tomatoes, carrots, cabbage, and strawberries. Each garden will have a fence around it to keep rabbits out. The tomato, carrot, and cabbage gardens will all be the same size. Since Jenna loves strawberries, the strawberry garden will be larger. It will use the same amount of fencing as the other three gardens combined. She used a total of about 113.04 feet of fencing material.

The area of the each smaller garden is about [ ] square feet.

The area of the larger garden is about [ ] square feet.

The area of the larger garden is [ ] times larger than the area of the smaller garden.

Submit

Explanation

First, write an equation for the total amount of fence. The amount of fence is equal to the sum of all the circumferences of the four gardens. Remember, the circumference of the larger garden is equal to the circumferences of the three smaller gardens combined. In this situation, let *r* represents the radius of the smaller circles.

$$3(\text{smaller circumference}) + \text{larger circumference} = 113.04 \text{ ft}$$
$$3(2\pi r) + 3(2\pi r) = 113.04 \text{ ft}$$
$$6(3.14)r + 6(3.14)r = 113.04 \text{ ft}$$
$$18.84r + 18.84r = 113.04 \text{ ft}$$
$$37.68r = 113.04 \text{ ft}$$
$$r = \frac{113.04 \text{ ft}}{37.68}$$
$$r = 3 \text{ ft}$$

Now, use the formula for area to find the area of each smaller garden.

$$A = \pi r^2$$
$$= (3.14)(3 \text{ ft})^2$$
$$= (3.14)(9 \text{ ft}^2)$$
$$= 28.26 \text{ ft}^2$$

So, the area of each smaller garden is **28.26** square feet.

Since the circumference of the larger garden is equal to the circumferences of the three smaller gardens combined, the circumference of the larger garden is equal to one-half of the total circumference, or 56.52 feet. Use this value and the circumference formula to find the radius of the larger garden.

larger garden.

$$C = 2\pi r$$
$$56.52 \text{ ft} = 2(3.14)r$$
$$56.52 \text{ ft} = 6.28r$$
$$\frac{56.52 \text{ ft}}{6.28} = r$$
$$9 \text{ ft} = r$$

Now, use the formula for area to find the area of the larger garden.

$$\begin{aligned} A &= \pi r^2 \\ &= (3.14)(9 \text{ ft})^2 \\ &= (3.14)(81 \text{ ft}^2) \\ &= 254.34 \text{ ft}^2 \end{aligned}$$

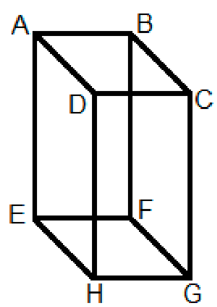
So, the area of the larger garden is **254.34** square feet.

To find out how many times larger the area of the larger garden is than the smaller gardens, divide.

$$254.34 \text{ ft}^2 \div 28.26 \text{ ft}^2 = 9$$

So, the area of the larger garden is **9** times larger than the area of the smaller garden.

Three-Dimensional Figures



Note: Figure is not drawn to scale.

In the rectangular prism above, *BF* = 21 units, *EF* = 12 units, and *BC* = 9 units.

If the rectangular prism is divided into two parts by a plane parallel to face DCGH, which of the following best describes the resulting cross-section of the prism?

A a rectangle with a length of 12 units and a width of 9 units

B a square with side lengths of 21 units

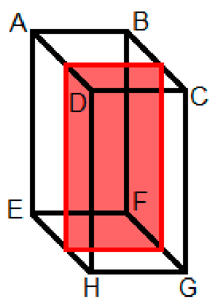
C a rectangle with a length of 21 units and a width of 9 units

D a rectangle with a length of 21 units and a width of 12 units

---

Explanation

Below, the rectangular prism is divided into two parts by a plane parallel to face DCGH.



The cross-section created by the plane is a rectangle with a length equal to *BF*, or 21 units. The width of the cross-section is equal to *EF*, or 12 units.

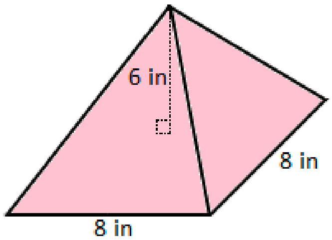
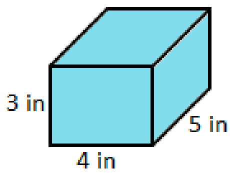
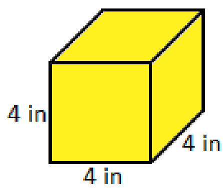
Therefore, if the rectangular prism is divided into two parts by a plane parallel to face DCGH, the resulting cross-section will be **a rectangle with a length of 21 units and a width of 12 units**.

. **Directions: Drag each tile to the correct location on the table. Each tile can be used more than once.**

Match the attributes to each three-dimensional shape that has the cross-section described.

Only consider cross-sections with whole number dimensions parallel to a side of one of the prisms or the base of the pyramid.

Quadrilateral A: area of 16 square inches	Quadrilateral B: perimeter of 24 inches	Quadrilateral C: perimeter of 16 inches

3-Dimensional Shape	Cross-sections
	
	
	

Explanation

The first shape is a square pyramid with a base of 8 inches by 8 inches. As a cross-section parallel to the base moves higher on the pyramid, the dimensions decrease but the cross-section always remains a square.

So, the cross-sections of this shape that fit these conditions are squares with whole number dimensions. The dimensions range from 1 inch by 1 inch to 7 inches by 7 inches.

Now, look at the description of quadrilateral A. There is a square in the range that has an area of 16 square inches: the square with dimensions of 4 inches by 4 inches. So, **the square pyramid does have quadrilateral A as a cross-section.**

Next, look at the description of quadrilateral B. There is a square in the range that has a perimeter of 24 inches: the square with dimensions of 6 inches by 6 inches. So, **the square pyramid does have quadrilateral B as a cross-section.**

Last, look at the description of quadrilateral C. There is a square in the range that has a perimeter of 16 inches: the square with dimensions of 4 inches by 4 inches. So, **the square pyramid does have quadrilateral C as a cross-section.**

The second shape is a rectangular prism with a length of 4 inches, a width of 5 inches, and a height of 3 inches. There are three possible dimensions for the cross-sections that are parallel to one side of the prism: 3 inches by 4 inches, 3 inches by 5 inches, and 4 inches by 5 inches.

Now, look at the description of quadrilateral A. None of the rectangles found have an area of 16 square inches. So, the rectangular prism does not have quadrilateral A as a cross-section.

Next, look at the description of quadrilateral B. None of the rectangles found have a perimeter of 24 inches. So, the rectangular prism does not have quadrilateral B as a cross-section.

Last, look at the description of quadrilateral C. There is a rectangle found that has a perimeter of 16 inches: the rectangle with dimensions of 3 inches by 5 inches. So, **the rectangular prism does have quadrilateral C as a cross-section.**

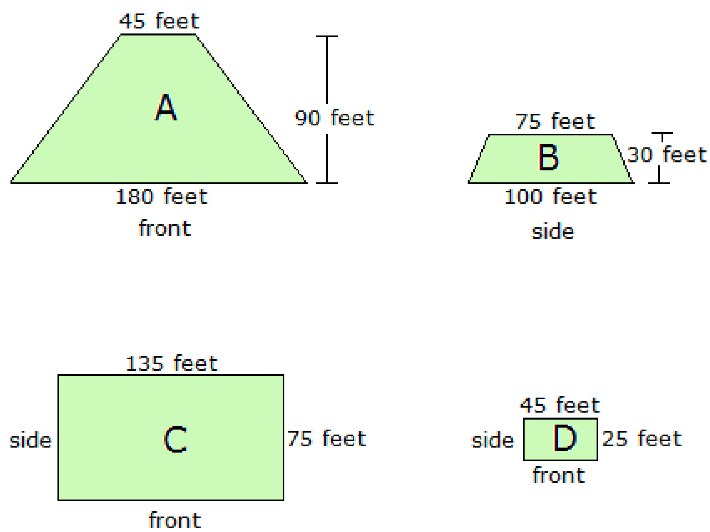
The third shape is a cube with side lengths of 4 inches. There is one possible dimension for the cross-sections that are parallel to one side of the cube: 4 inches by 4 inches.

Now, look at the description of quadrilateral A. The square found does have an area of 16 square inches. So, **the cube does have quadrilateral A as a cross-section.**

Next, look at the description of quadrilateral B. The square found does not have a perimeter of 24 inches. So, the cube does not have quadrilateral B as a cross-section.

Last, look at the description of quadrilateral C. The square found does have a perimeter of 16 inches. So, **the cube does have quadrilateral C as a cross-section.**

. **Directions: Select the correct answer from each drop-down menu.**



The cross sections shown above are from a right rectangular pyramid.

Cross section A is from a plane that is perpendicular to the base and parallel to the front of the base of the pyramid.

Cross section B is from a plane that is perpendicular to the base and parallel to the sides of the base of the pyramid.

Cross section C is parallel to the base and intersects the pyramid at $\frac{1}{4}$ of the way up.

Cross section D is parallel to the base and intersects the pyramid at $\frac{3}{4}$ of the way up.

The pyramid from which the cross sections were taken has a front length of [▾] feet, a side width of [▾] feet, and a height of [▾] feet.

Explanation

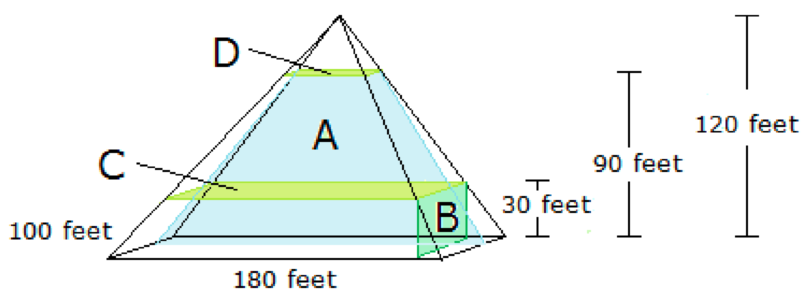
Cross section A is perpendicular to the base of the pyramid, which indicates that one side of the cross section represents a part of the base. Since the cross section is parallel to the front of the base of the pyramid, the longest side of the cross section will give the length across the front of the pyramid. Therefore, the pyramid has a front length of **180** feet.

Cross section B is perpendicular to the base of the pyramid, which indicates that one side of the cross section represents a part of the base. Since the cross section is parallel to the sides of the base of the pyramid, the longest side of the cross section will give the width across the side of the pyramid. Therefore, the pyramid has a side width of **100** feet.

Cross section C is parallel to the base, which means it is a scaled version of the base. Since the cross section is also $\frac{1}{4}$ of the way up the pyramid, determine if this can be used to find the height of the pyramid. Notice that the side of cross section C is the same length as the top of cross section B. Since cross section B is cutting through the side, this indicates that cross section C is 30 feet up from the base of the pyramid. If 30 feet is $\frac{1}{4}$ of the height of the pyramid, then the height of the pyramid is 4×30 feet = 120 feet.

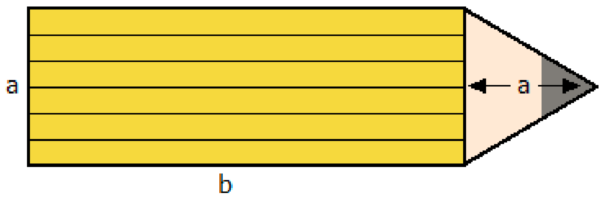
Similarly, cross section D is parallel to the base, which means it is a scaled version of the base. Since the cross section is also $\frac{3}{4}$ of the way up the pyramid, determine if this can be used to find the height of the pyramid. Notice that the side of cross section D is the same length as the top of cross section A. Since cross section A is cutting through the front, this indicates that cross section D is 90 feet up from the base of the pyramid. If 90 feet is $\frac{3}{4}$ of the height of the pyramid, then the height of the pyramid is $\frac{4}{3} \times 90$ feet = 120 feet.

Using either cross section C or D, it can be concluded that the height of the pyramid is **120** feet.



Area, Surface Area, and Volume

1. Ms. Hatcher bought the pencil decoration shown below for her bulletin board.



Note: figure not drawn to scale

If $a = 10$ in and $b = 25$ in, what is the area of the pencil decoration?

$$\text{Area of a triangle} = \frac{1}{2}bh$$
$$\text{Area of a rectangle} = lw$$

- **A** 300 in^2
- **B** 350 in^2
- **C** 250 in^2
- **D** 125 in^2

Explanation

Since this is an unusual shape, divide it up into a rectangle and a triangle.

First, find the area of the rectangle and the area of the triangle.

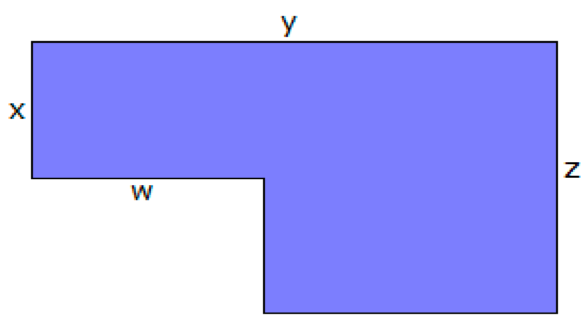
$$\begin{aligned} \text{Area}_{\text{rect}} &= lw \\ &= ba \\ &= (25\text{ in})(10\text{ in}) \\ &= 250\text{ in}^2 \end{aligned}$$

$$\begin{aligned} \text{Area}_{\text{tri}} &= \frac{1}{2}bh \\ &= \frac{1}{2}a \cdot a \\ &= \frac{1}{2}(10\text{ in})(10\text{ in}) \end{aligned}$$

$$= 50 \text{ in}^2$$

Next, add the area of the rectangle and the area of the triangle to find the area of the pencil decoration.

$$\text{Area}_{\text{rect}} + \text{Area}_{\text{tri}} = 250 \text{ in}^2 + 50 \text{ in}^2$$
$$= 300 \text{ in}^2$$



Note: Figure is not drawn to scale.

If x = 3 inches, y = 11 inches, w = 5 inches, and z = 5 inches, what is the area of the object?

$$\boxed{\text{Area of a rectangle} = lw}$$

A 35 square inches

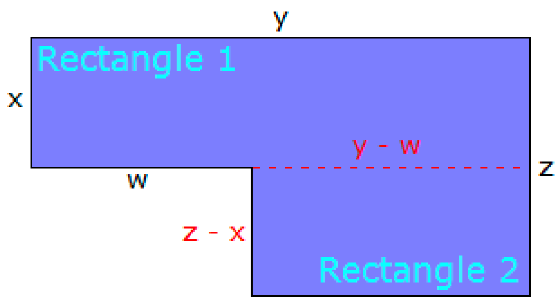
B 45 square inches

C 24 square inches

D 66 square inches

Explanation

To find the area of the object, divide it into two rectangles, as shown below.



First, find the area of rectangle 1.

$$\begin{aligned} \text{Area}_{\text{rect 1}} &= lw \\ &= yx \\ &= (11 \text{ in})(3 \text{ in}) \\ &= 33 \text{ in}^2 \end{aligned}$$

Next, find the area of rectangle 2.

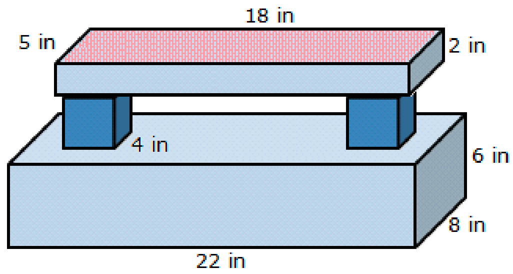
$$\begin{aligned} \text{Area}_{\text{rect 2}} &= lw \\ &= (y - w)(z - x) \\ &= (11 \text{ in} - 5 \text{ in})(5 \text{ in} - 3 \text{ in}) \\ &= (6 \text{ in})(2 \text{ in}) \\ &= 12 \text{ in}^2 \end{aligned}$$

Now, add the areas of the two rectangles to find the total area of the object.

$$\begin{aligned} \text{Total area} &= 33 \text{ in}^2 + 12 \text{ in}^2 \\ &= 45 \text{ in}^2 \end{aligned}$$

Therefore, the area of the object is **45 square inches**.

3. **Directions: Type the correct answer in each box. Use numerals instead of words. If necessary, use / for the fraction bar(s).**



Note: figure not drawn to scale.

Seth built a wooden step stool out of two rectangular prisms and two cubes. Before fastening the components together, he stained each component to give it its final color.

To completely stain the bottom rectangular prism, Seth had to cover ____ square inches of wood. For one of the cubes to be completely stained, he had to cover ____ square inches of wood. To completely stain the top rectangular prism, he had to cover ____ square inches of wood.

Once the stool was fastened together, he applied a coat of sealant to all exposed surfaces of the stool, including the bottom, to cover a total of ____ square inches.

Explanation

The rectangular prism that makes up the bottom of the stool has a length of 22 inches, width of 8 inches, and height of 6 inches. Use the formula for the surface area of a rectangular prism to find its surface area.

$$\begin{aligned} SA &= 2(l \times w) + 2(l \times h) + 2(w \times h) \\ &= 2(22 \text{ in} \times 8 \text{ in}) + 2(22 \text{ in} \times 6 \text{ in}) + 2(8 \text{ in} \times 6 \text{ in}) \\ &= 352 \text{ in}^2 + 264 \text{ in}^2 + 96 \text{ in}^2 \\ &= 712 \text{ in}^2 \end{aligned}$$

Therefore, to completely stain the bottom rectangular prism Seth will have to cover **712** square inches of wood.

Each cube that is between the two rectangular prisms has a side length of 4 inches. Use the formula for the surface area of a cube to find its surface area.

$$\begin{aligned} SA &= 6(l \times l) \\ &= 6(4 \text{ in} \times 4 \text{ in}) \\ &= 96 \text{ in}^2 \end{aligned}$$

Therefore, to completely stain one of the cubes Seth will have to cover **96** square inches of wood.

The rectangular prism that makes up the top step of the stool has a length of 18 inches, width of 5 inches, and height of 2 inches. Use the formula for the surface area of a rectangular prism to find its surface area.

$$\begin{aligned} SA &= 2(l \times w) + 2(l \times h) + 2(w \times h) \\ &= 2(18 \text{ in} \times 5 \text{ in}) + 2(18 \text{ in} \times 2 \text{ in}) + 2(5 \text{ in} \times 2 \text{ in}) \\ &= 180 \text{ in}^2 + 72 \text{ in}^2 + 20 \text{ in}^2 \\ &= 272 \text{ in}^2 \end{aligned}$$

Therefore, to completely stain the top rectangular prism Seth will have to cover **272** square inches of wood.

Find the combined surface area of the individual pieces that make up the stool.

$$712 \text{ in}^2 + 96 \text{ in}^2 + 96 \text{ in}^2 + 272 \text{ in}^2 = 1{,}176 \text{ in}^2$$

After the stool is fastened together, in each place where a cube is attached to a rectangular prism, neither that side of the cube nor the corresponding area on the rectangular prism will be exposed. So, for each contact between a cube and a rectangular prism $2 \times 16 \text{ in}^2$ needs to be subtracted from the sum of the surface areas. Since there are four places where they contact, a total of $4(2 \times 16 \text{ in}^2)$ needs to be subtracted from the combined surface areas to get the surface area of the stool after it has been fastened together.