7.4 MATH PATHWAYS

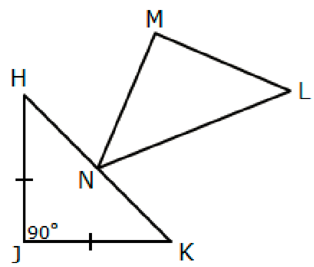
a. ANGLES

The following facts can be applied to find an unknown angle in a figure.

- *Two angles are **complementary** if the sum of their measures is 90°.*
- *Two angles are **supplementary** if the sum of their measures is 180°.*
- *If two angles are supplementary, then they form a **linear pair**.*
- *Angles that share a vertex and a side are called **adjacent angles**.*
- *If multiple adjacent angles form a **straight angle**, then the sum of their measures is 180°.*
- *Opposite angles formed by two intersecting lines are called **vertical angles**.*
- ***Vertical angles** are always congruent.*
- *The sum of the angle measures in a triangle always equals **180°**.*
- *The sum of the angle measures in a quadrilateral always equals **360°**.*

Example 1:

In the figure below, triangle HJK is congruent to triangle LMN.



If m∠LNK = 66°, what is m∠HNM?

Solution:

Since triangle HJK is an isosceles triangle, its base angles are congruent. So, m∠JHK = m∠JKH.

The sum of the angle measures in a triangle always equals 180°.

$$\begin{aligned} m\angle HJK + m\angle JHK + m\angle JKH &= 180^\circ \\ 90^\circ + 2(m\angle JKH) &= 180^\circ \\ 2(m\angle JKH) &= 90^\circ \\ m\angle JKH &= 45^\circ \end{aligned}$$

Since triangle HJK is congruent to triangle LMN, their corresponding angles are congruent. So, $\angle JKH \cong \angle MNL$ and $m\angle MNL = 45°$.

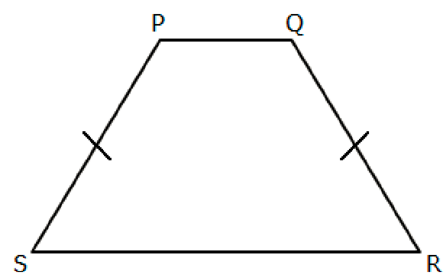
In the figure, ∠HNM, ∠MNL, and ∠LNK form a straight angle; therefore, the sum of their measures is 180°.

$$\begin{aligned} m\angle HNM + m\angle MNL + m\angle LNK &= 180° \\ m\angle HNM + 45° + 66° &= 180° \\ m\angle HNM + 111° &= 180° \\ m\angle HNM &= 69° \end{aligned}$$

Therefore, $m\angle HNM =$ **69°**.

Example 2:

In the figure below, trapezoid PQRS is an isosceles trapezoid with side PS congruent to side QR.



If $m\angle QRS = 59°$, what is $m\angle SPQ$?

Solution:

Since trapezoid PQRS is an isosceles trapezoid, its base angles are congruent. So, $m\angle QRS = m\angle PSR$ and $m\angle RQP = m\angle SPQ$.

The sum of the angle measures in a quadrilateral always equals 360°.

$$\begin{aligned} m\angle QRS + m\angle PSR + m\angle RQP + m\angle SPQ &= 360° \\ 2(m\angle QRS) + 2(m\angle SPQ) &= 360° \\ 2(59°) + 2(m\angle SPQ) &= 360° \\ 118° + 2(m\angle SPQ) &= 360° \\ 2(m\angle SPQ) &= 242° \\ m\angle SPQ &= 121° \end{aligned}$$

Therefore, $m\angle SPQ =$ **121°**.

VIDEOS:

Lines and Angles: https://www.khanacademy.org/math/cc-seventh-grade-math/cc-7th-geometry/cc-7th-angles/v/angles-at-the-intersection-of-two-lines?v=_7aUxFzTG5w

Triangle Angle: https://www.khanacademy.org/math/geometry/parallel-and-perpendicular-lines/triang_prop_tut/v/triangle-angle-example-1

b. TRIANGLES

A triangle is a polygon with 3 sides.
A triangle can be named by the measure of its sides or angles.

Triangles by Sides

Equilateral triangles have 3 sides of equal length and 3 equal angles measuring 60°.

Isosceles triangles have at least 2 sides of equal length. *An equilateral triangle is a special case of an isosceles triangle.*

Scalene triangles have NO sides of equal length.

Triangles by Angles

Right triangles have 1 angle that measures 90°.

Acute triangles have 3 angles that measure less than 90°.

Obtuse triangles have 1 angle that measures more than 90°.

Equilateral Triangle	Isosceles Triangle	Scalene Triangle	Right Triangle	Acute Triangle	Obtuse Triangle
3 equal sides	2 equal sides	0 equal sides	1 right angle	3 acute angles	1 obtuse angle

*A **triangle** has three angle measures that add up to 180°. A triangle also has three side lengths such that the sum of any two sides is greater than the third side.*

Example 1:

How many triangles exist with the given angle measures?

10°, 50°, 120°

Solution:

Since the sum of the given angles is 180°, the angles do form a triangle.

Since all three angles are different, the triangle is a scalene triangle. An infinite number of scalene triangles exist with different side lengths.

Therefore, **more than one triangle exists with the given angle measures**.

Example 2:

How many triangles exist with the given angle measures?

42°, 51°, 85°

Solution:

Since the sum of the given angles is 178°, the angles do not form a triangle.

Therefore, **no triangle exists with the given angle measures**.

Example 3:

How many triangles exist with the given side lengths?

17 cm, 17 cm, 35 cm

Solution:

According to the Triangle Inequality theorem, the sum of any two sides must be greater than the third side.

Since the sum of 17 cm and 17 cm is less than 35 cm, the triangle does not exist.

Therefore, **no triangle exists with the given side lengths**.

Example 4:

How many triangles exist with the given side lengths?

3 feet, 3 feet, 3 feet

Solution:

A triangle with three equal sides is an equilateral triangle. Only one equilateral triangle with side lengths of 3 feet exists.

Therefore, **exactly one unique triangle exists with the given side lengths**.

c. CIRCLES

Circumference & Area of Circles

VOCABULARY

Diameter
the length of a line segment from one side of a circle to the other, passing through the center of the circle

Radius
the length of a line segment from the center of a circle to its edge; the radius is half the value of the diameter

Circumference
the distance around a circle

FORMULAS

Circumference (C) = $2\pi r$ *or* πd, *where* r *is the radius and* d *is the diameter*

Area (A) = πr^2, *where* r *is the radius*

Note: An approximation of pi (π *) is often given as 3.14.*

Example 1:

A circle has a radius of 14 inches. What is the circumference of the circle? Use 3.14 for π.

Solution:

Substitute *r* = 14 inches into the circumference formula, and solve for *C*.

$$\begin{aligned} C &= 2\pi r \\ &= 2(3.14)(14 \text{ inches}) \\ &= 87.92 \text{ inches} \end{aligned}$$

Example 2:

A circle has a circumference of 44 cm. What is the area of the circle? Use 3.14 for π.

Solution:

First, use the formula for circumference to find the radius.

$$\begin{aligned} C &= 2\pi r \\ 44 \text{ cm} &= 2(3.14)(r) \\ 7 \text{ cm} &\approx r \end{aligned}$$

Next, use the value of the radius in the formula for area.

$$\begin{aligned} A &= \pi r^2 \\ &= (3.14)(7 \text{ cm})^2 \\ &= (3.14)(49 \text{ cm}^2) \\ &= 154 \text{ cm}^2 \end{aligned}$$

VIDEO:

Circles: Radius, diameter, circumference, and Pi https://www.khanacademy.org/math/geometry/cc-geometry-circles/circles/v/circles-radius-diameter-and-circumference?v=jyLRpr2P0MQ

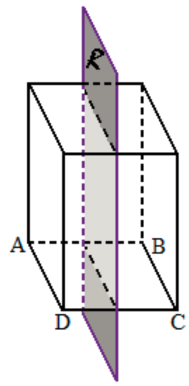
Area of Circle: https://www.khanacademy.org/math/algebra-basics/core-algebra-foundations/alg-foundations-circum_area_circles/v/area-of-a-circle?v=ZyOhRgnFmIY

d. Three-Dimensional Figures

Plane Intersections

*The two-dimensional figure obtained by a solid's intersection with a plane is referred to as a **cross-section**.*

Shown below is the intersection of a plane with a rectangular prism, where face *ABCD* is a square.



Plane R is perpendicular to face ABCD.

Example:

Which two-dimensional shape depicts the cross-section shown above?

Solution:

The plane that intersects the rectangular prism is parallel to a rectangular face. Therefore, the cross-section is a **rectangle**.

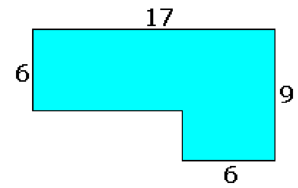
If the plane had intersected the rectangular prism parallel to the square face, the cross-section would have been a square.

e. Area, Surface, and Volume

*To find the **area** of composite shapes, break the shape into smaller shapes whose areas are easier to find.*

Example:

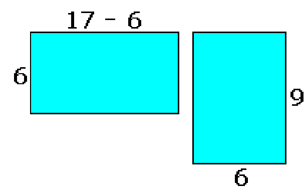
Find the area of the following shape.



(measurements in inches)

Answer:

To find the area of this shape, first break it up into rectangles.



Now, find the area of each rectangle.

area of left rectangle	=	6 in × (17 in - 6 in)
	=	6 in × 11 in
	=	66 in^2
area of right rectangle	=	6 in × 9 in
	=	54 in^2

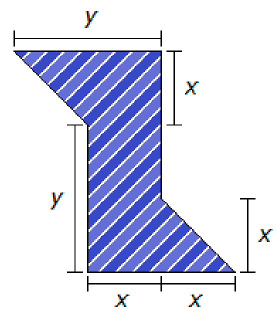
Finally, add the two areas together.

Total area	=	66 in^2 + 54 in^2
	=	**120 in^2**

*To find the **area** of a composite figure, break the figure up into smaller shapes.*

Example 1:

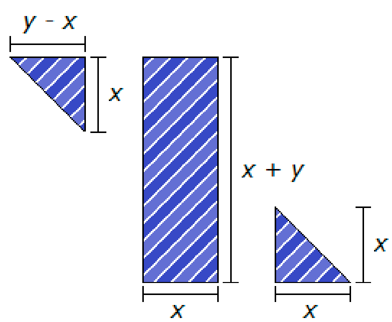
Zane designed the T-shirt logo shown below.



If $x = 4$ inches and $y = 8$ inches, what is the area of the logo?

Solution:

Since this is an unusual shape, break it up into a rectangle and two triangles.



Now, find the area of the rectangle and the area of the two triangles.

$$\begin{aligned} A_{\text{rectangle}} &= (\text{length})(\text{width}) \\ &= (x + y)(x) \\ &= (4\text{ in} + 8\text{ in})(4\text{ in}) \\ &= (12\text{ in})(4\text{ in}) \\ &= 48\text{ in}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{top triangle}} &= \tfrac{1}{2}(\text{base})(\text{height}) \\ &= \tfrac{1}{2}(y - x)(x) \\ &= \tfrac{1}{2}(8\text{ in} - 4\text{ in})(4\text{ in}) \\ &= \tfrac{1}{2}(4\text{ in})(4\text{ in}) \\ &= 8\text{ in}^2 \end{aligned}$$

$$
\begin{aligned}
A_{\text{bottom triangle}} &= \tfrac{1}{2}(\text{base})(\text{height}) \\
&= \tfrac{1}{2}(x)(x) \\
&= \tfrac{1}{2}(4 \text{ in})(4 \text{ in}) \\
&= 8 \text{ in}^2
\end{aligned}
$$

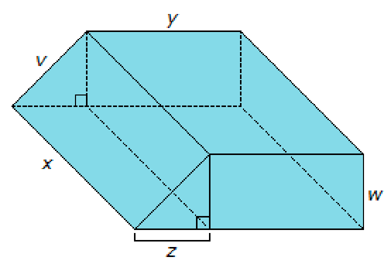
Finally, add the areas of the rectangle and the two triangles.

$$
\begin{aligned}
\text{Area} &= 48 \text{ in}^2 + 8 \text{ in}^2 + 8 \text{ in}^2 \\
&= 64 \text{ in}^2
\end{aligned}
$$

*To find the **surface area** of a composite figure, calculate the sum of the outside areas of the faces that compose the figure.*

Example:

The display case below is composed of a rectangular prism and a triangular prism.



**Note: Figure is not drawn to scale.*

If v = 15 in, w = 12 in, x = 34 in, y = 22 in, and z = 9 in, what is the surface area of the display case?

Solution:

To find the surface area of the display case, find the sum of the areas of the outside faces of the rectangular prism and the triangular prism.

First, find the outside areas of the rectangular prism.

$$
\begin{aligned}
A_{\text{front and back}} &= 2(\text{length})(\text{width}) \\
&= 2yw \\
&= 2(22 \text{ in})(12 \text{ in}) \\
&= 528 \text{ in}^2
\end{aligned}
$$

$$
\begin{aligned}
A_{\text{top and bottom}} &= 2(\text{length})(\text{width}) \\
&= 2yx \\
&= 2(22 \text{ in})(34 \text{ in}) \\
&= 1{,}496 \text{ in}^2
\end{aligned}
$$

$$
\begin{aligned}
A_{\text{side}} &= (\text{length})(\text{width}) \\
&= xw \\
&= (34 \text{ in})(12 \text{ in}) \\
&= 408 \text{ in}^2
\end{aligned}
$$

Add those areas.

$$528 \text{ in}^2 + 1{,}496 \text{ in}^2 + 408 \text{ in}^2 = 2{,}432 \text{ in}^2$$

Next, find the outside areas of the triangular prism.

$$\begin{aligned} A_{\text{triangles}} &= 2\left(\frac{1}{2}\right)(\text{base})(\text{height}) \\ &= 2\left(\frac{1}{2}\right)zw \\ &= 2\left(\frac{1}{2}\right)(9 \text{ in})(12 \text{ in}) \\ &= 108 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{slanted rectangle}} &= (\text{length})(\text{width}) \\ &= xv \\ &= (34 \text{ in})(15 \text{ in}) \\ &= 510 \text{ in}^2 \end{aligned}$$

$$\begin{aligned} A_{\text{bottom rectangle}} &= (\text{length})(\text{width}) \\ &= xz \\ &= (34 \text{ in})(9 \text{ in}) \\ &= 306 \text{ in}^2 \end{aligned}$$

Add those areas.

$$108 \text{ in}^2 + 510 \text{ in}^2 + 306 \text{ in}^2 = 924 \text{ in}^2$$

To find the total surface area, find the sum of the outside areas of the rectangular prism and the triangular prism.

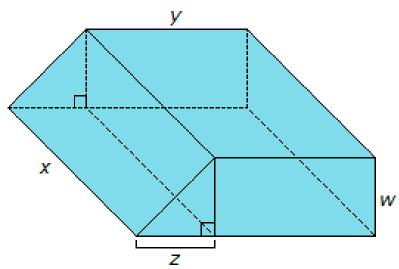
$$2{,}432 \text{ in}^2 + 924 \text{ in}^2 = 3{,}356 \text{ in}^2$$

So, the total surface area of the display case is **3,356 in^2**.

*To find the **volume** of a composite figure, find the sum of the individual volumes of the solids that compose the figure.*

Example:

The display case below is composed of a rectangular prism and a triangular prism.



**Note: Figure is not drawn to scale.*

If w = 12 in, x = 34 in, y = 22 in, and z = 10 in, what is the volume of the display case?

Solution:

To find the volume of the display case, find the sum of the volumes of the rectangular prism and the triangular prism.

First, find the volume of the rectangular prism.

The formula for the volume of the rectangular prism is given below.

$$\begin{aligned} V_{\text{rectangular prism}} &= (\text{length})(\text{width})(\text{height}) \\ &= xyw \\ &= (34 \text{ in})(22 \text{ in})(12 \text{ in}) \\ &= 8{,}976 \text{ in}^3 \end{aligned}$$

Next, find the volume of the triangular prism.

The formula for the volume of the triangular prism is given below.

$$\begin{aligned} V_{\text{triangular prism}} &= (\text{area of the base})(\text{height}) \\ &= \tfrac{1}{2}zwx \\ &= \tfrac{1}{2}(10 \text{ in})(12 \text{ in})(34 \text{ in}) \\ &= 2{,}040 \text{ in}^3 \end{aligned}$$

Now, find the sum of the two volumes.

$$\begin{aligned} V_{\text{display case}} &= V_{\text{rectangular prism}} + V_{\text{triangular prism}} \\ &= 8{,}976 \text{ in}^3 + 2{,}040 \text{ in}^3 \\ &= 11{,}016 \text{ in}^3 \end{aligned}$$

Therefore, the volume of the display case is **11,016 in^3**.